Section 6.6 Exponential and Logarithmic Equations

Properties of Logarithms with base b, where $b, x, y > 0, b \neq 1$, and p is a real number

1. $\log_b 1 = 0$
   $b^0 = 1$
2. $\log_b b = 1$
   $b^1 = b$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x$

5) $\log_b(x \cdot y) = \log_b x + \log_b y$ — product property

6) $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$ — quotient property

7) $\log_b(x^p) = p \cdot \log_b x$ — power property

Solving Exponential Equations

- isolate the $\boxed{\text{exponential expression}}$
- take the log of both sides
- simplify using log properties
- solve the equation by isolating the $\boxed{\text{variable}}$

$x, t \ldots$

1) $5^{2x} = 3$

$\log 5^{2x} = \log 3$ power property

$\frac{2x \cdot \log 5}{2 \log 5} = \frac{\log 3}{2 \log 5}$

$\boxed{x = \frac{\log 3}{2 \log 5}}$ exact answer

$\boxed{x \approx 0.341}$ approx answer

Solving Exponential Equations

1. $5^{2x} = 3$

4. $e^{x^2+x} = e^{3x+15}$

2. $160e^{-0.03t} = 80$ $\quad | \div 160$

$e^{-0.03t} = \frac{80}{160}$

5. $(1 + 0.05/4)^{4t} = 3$

3. $96 - 10^x = 12$

$e^{-0.03t} = \frac{1}{2}$ $\quad |$ take ln both sides

$\ln e^{-0.03t} = \ln \frac{1}{2}$

$-0.03t \ln e = \ln(1/2)$

$-0.03t = \ln(1/2)$ $\quad | \div -0.03$

$\boxed{t = \frac{\ln(1/2)}{-0.03}}$

$\boxed{t = 23.105}$

5. $\log\left(1+\frac{0.05}{4}\right)^{4t} = \log 3$

$4t \log\left(1+\frac{.05}{4}\right) = \log 3$

$\boxed{t = \frac{\log 3}{4 \log\left(1+\frac{.05}{4}\right)}}$

(3) $96 - 10^x = 12$

$-10^x = 12 - 96$

$-10^x = -84$

$10^x = 84$

$\log 10^x = \log 84$

$x \log 10 = \log 84$

$\boxed{x = \log 84}$

(4) $e^{x^2+x} = e^{3x+15}$

$x^2 + x = 3x + 15$

$x^2 - 2x - 15 = 0$

$(x+3)(x-5) = 0$

$x = -3, \; x = 5$

Solving log equations (all terms have a log with the same base)

1. condense logs into a single log (first use prop 7), if multiple logs on one side
2. solve the equation using the arguments
3. check and discard extraneous solutions

EX Solve the logarithmic equation algebraically.

1. $\log(2x+4) = \log(3-x)$

$2x+4 = 3-x$

$3x = -1 \Rightarrow \boxed{x = -1/3}$ ✓

check:

$\log(2(-1/3)+4) \overset{?}{=} \log(3-(-1/3))$

$\log(-\frac{2}{3}+4) \overset{?}{=} \log(3+1/3)$

$\log \frac{10}{3} \overset{?}{=} \log \frac{10}{3}$ ✓

2. $\ln(x) + \ln(x-3) = \ln(5x-7)$

$\ln(x(x-3)) = \ln(5x-7)$

$x(x-3) = 5x-7$

$x^2-3x-5x+7 \Rightarrow x^2-8x+7=0$

$(x-7)(x-1)=0$

$\boxed{x=7}$ or $\cancel{\boxed{x=1}}$

check $x=7$

$\ln 7 + \ln(7-3) \overset{?}{=} \ln(5\cdot 7-7)$

$\ln 7 + \ln 4 \overset{?}{=} \ln 28$

$\ln 28 = \ln 28$

check $x=1$

$\ln 1 + \ln(1-3) \overset{?}{=} \ln(5\cdot 1-7)$

3. $2\log_5 x = \log_5(7) + \log_5(3x)$

$\log_5 x^2 = \log_5(7\cdot 3x)$

$x^2 = 21x$

$x^2 - 21x = 0$

$x(x-21) = 0$

$\cancel{x=0}$ not

$x = 21$ ✓

Solving log equations (has a term without a log)

1. isolate terms with a log of same base to one side
2. condense logs into a single log and isolate
3. write as an exponential equation
4. solve the equation
5. check and discard extraneous solutions

4. $-3 + 4\ln(7x) = 2$

$4\ln(7x) = 5$

$\boxed{\ln(7x) = \frac{5}{4}}$

$\log_e 7x = 5/4$

$e^{5/4} = 7x$

$x = \frac{e^{5/4}}{7}$

$-3 + 4\ln \cancel{7} \cdot \frac{e^{5/4}}{\cancel{7}} =$

$= -3 + 4\ln e^{5/4}$

$= -3 + 4 \cdot \frac{5}{4} \ln e$

$= -3 + 5 = 2$

5. $\ln(\sqrt{x-8}) = 5$

$e^5 = \sqrt{x-8}$ $/ \wedge 2$

$e^{10} = x - 8 \Rightarrow x = e^{10} + 8$

6. $3\log_2(x+4) = 7$

$\log_2(x+4) = 7/3$

$2^{7/3} = x + 4 \Rightarrow x = 2^{7/3} - 4$

7. $\log_4(x+2) = 1 - \log_4(x-1)$

$+\log_4(x-1)$ $\quad +\log_4(x-1)$

$\log_4(x+2) + \log_4(x-1) = 1$

$\log_4 (x+2)(x-1) = 1$

$4^1 = (x+2)(x-1)$

$x^2 - x + 2x - 2 - 4 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ $\quad x = 2$

8. $\log x = 2 + \log(x-3)$

$\log x - \log(x-3) = 2$

$\log \frac{x}{x-3} = 2$

$10^2 = \frac{x}{x-3} \Rightarrow 100 = \frac{x}{x-3} \Rightarrow 100(x-3) = x$

$100x - 300 - x = 0$

$99x = 300 \Rightarrow x = \frac{300}{99}$