MTH 122 — Exam 3 Study Guide

College of Lake County
Spring 2026

Use this guide to review concepts, locate your notes, and practice each problem.
*Fill in every blank **before** you attempt the problem.*

Name: _____

How to use this guide: For each problem, answer the reflection questions first — topic, -2notes page, and needed formulas. Then solve the problem in the work space provided. This is how you *study*, not just practice!

Problem 1 (5 points)

The Problem: Use long division to divide. Express any numbers as integers or simplified fractions.

$$(x^5 + 2x^4 + 12x^2 - 5x + 16) \div (x^2 + 2)$$

Study Prompts — Answer these before solving!

1. **Topic:** What concept/topic does this problem cover?

   _____

2. **Notes Check:** What page or section of your notes covers polynomial long division?

   Page(s)/Section: _____

3. **Setup:** When setting up polynomial long division, what must you do if a power of x is *missing* from the dividend?

   _____

   _____

4. **Process:** In your own words, write the steps for polynomial long division (Divide → Multiply → Subtract → Bring Down):

   D: _____

   M: _____

   S: _____

   B: _____

5. **Final Answer Form:** How do you write the final answer when there is a remainder R?

   _____

Your Work:

Problem 2 (3 points)

The Problem: Given a polynomial $f(x)$, the quotient $\dfrac{f(x)}{x-8}$ has a remainder of 25. What is the value of $f(8)$?

Study Prompts — Answer these before solving!

1. **Topic:** What theorem does this problem directly apply?

   _____

2. **Notes Check:** What page or section of your notes covers this theorem?

   Page(s)/Section: _____

3. **State the Theorem:** Write the *Remainder Theorem* in your own words or as a formula:

   _____

   _____

4. **Connect to the Problem:** If $\dfrac{f(x)}{x-8}$ has a remainder of 25, what does the Remainder Theorem tell you about $f(8)$? Explain before you answer.

   _____

   _____

Your Work:

Problem 3 (3 points)

The Problem: Use the Factor Theorem to determine if $x - 4$ is a factor of $f(x) = x^3 + 64$.

Study Prompts — Answer these before solving!

1. **Topic:** What theorem does this problem use, and how is it related to Problem 2?

   _____

   _____

2. **Notes Check:** What page or section of your notes covers the Factor Theorem?

   Page(s)/Section: _____

3. **State the Theorem:** Write the *Factor Theorem* as a complete statement (both directions):

   - $(x-c)$ is a factor of $f(x)$ **if and only if** _____

4. **Test Value:** To test whether $x - 4$ is a factor, what value of x must you substitute into $f(x)$?

   Test value: _____

5. **Decision Rule:** After you evaluate f at that value, how do you decide whether $(x - 4)$ is or is not a factor?

   _____

   _____

Your Work:

Problem 4 (4 points)

The Problem: List all possible rational zeros of $n(x) = -4x^5 - 13x^3 + 6x^2 - 10$.

Study Prompts — Answer these before solving!

1. **Topic:** What theorem is used to generate a list of possible rational zeros?

   _____

2. **Notes Check:** What page or section of your notes covers this?

   Page(s)/Section: _____

3. **State the Theorem:** Write the Rational Zero Theorem formula. Label clearly what p and q represent.

   Possible rational zeros $= \frac{p}{q}$, where:

   $p =$ _____

   $q =$ _____

4. **Identify the Values:** For this polynomial, list the factors:

   - Factors of the constant term: _____
   - Factors of the leading coefficient: _____

5. **Reminder:** Don't forget to include both _____ and _____ signs for each ratio.

Your Work:

Problem 5 (3 points)

The Problem: If $f(x)$ is a polynomial with *real coefficients* and zeros of 5 (mult. 1), -5 (mult. 1), $-5+7i$, and $4-3i$, what is the minimum degree of $f(x)$?

Study Prompts — Answer these before solving!

1. **Topic:** What theorem about complex zeros of real polynomials is essential here?

   _____

2. **Notes Check:** What page or section of your notes covers complex/imaginary zeros and conjugate pairs?

   Page(s)/Section: _____

3. **State the Key Rule:** Write the *Complex Conjugate Zeros Theorem- We said in class that complex zeros come in pairs. This is called the Conjugate Theorem* in your own words:

   _____

   _____

4. **Apply It:** Given $-5+7i$ is a zero, what other zero *must* also exist?

   Additional zero: _____

   Given $4-3i$ is a zero, what other zero *must* also exist?

   Additional zero: _____

5. **Count:** List ALL zeros (including those forced by the theorem), count them, and state the minimum degree:

   All zeros: _____

   Minimum degree: _____

Your Work:

Problem 6 (6 points)

The Problem:

$$f(x) = x^4 - 16x^3 + 78x^2 + 32x - 160$$

Given that $8 - 4i$ and $\sqrt{2}$ are zeros, (a) find all zeros in exact form, and (b) factor $f(x)$ as a product of linear factors.

Study Prompts — Answer these before solving!

1. **Topic:** What two theorems do you need for this problem?

   Theorem 1: _____

   Theorem 2: _____

2. **Notes Check:** Page(s)/Section: _____

3. **Conjugate Zeros:** Based on the given zeros, write the two *additional* zeros the theorems force:

   - Conjugate of $8 - 4i$: _____
   - Conjugate of $\sqrt{2}$: _____

4. **Linear Factor Form:** Once you have all zeros r_1, r_2, r_3, r_4, how do you write $f(x)$ as a product of linear factors?

   $f(x) = a(\quad)(\quad)(\quad)(\quad)$ where $a =$ _____

Your Work:

Problem 7 (6 points)

The Problem: Consider the rational function

$$f(x) = \frac{(x+3)(1-6x)}{(5x+1)(x+3)}.$$

(a) Does it have a hole? Give coordinates.
(b) Does it have a horizontal asymptote? Give equation.
(c) Does it have vertical asymptote(s)? Give equation(s).

Study Prompts — Answer these before solving!

1. **Topic:** What features of rational functions are being analyzed here?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **Holes:** In your own words, when does a rational function have a *hole*? How do you find the coordinates of the hole?

   A hole occurs when: _____

   To find the x-coordinate: _____

   To find the y-coordinate: _____

4. **Vertical Asymptotes:** After canceling common factors, how do you find vertical asymptotes?

   _____

   _____

5. **Horizontal Asymptotes — Write all 3 cases:**

   - If degree of numerator $<$ degree of denominator: _____
   - If degree of numerator $=$ degree of denominator: _____
   - If degree of numerator $>$ degree of denominator: _____

6. **Identify the Degrees:** What is the degree of the numerator? _____ Denominator? _____
   Which case applies? _____

Your Work:

Problem 8 (8 points)

The Problem: Graph the rational function

$$f(x) = \frac{x^3 - x^2 - 2x}{x^3 + 5x^2 + 6x}.$$

Label all holes, x- and y-intercepts, and asymptotes.

Study Prompts — Answer these before solving!

1. **Topic:** What is the overall topic? What must you do *first* before analyzing this function?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **Graphing Checklist — Write the method for finding each feature:**

   - **Step 1 — Factor:** Factor numerator and denominator completely.
   - **Step 2 — Holes:** _____
   - **Step 3 — Vertical Asymptotes:** _____
   - **Step 4 — Horizontal Asymptote:** Compare degrees. Degrees here: num = _____, den = _____
     Result: _____
   - **Step 5 — x-intercept(s):** Set _____ $= 0$, solve for x (from simplified form).
   - **Step 6 — y-intercept:** Substitute $x =$ _____ into the _____ form.

4. **Pre-work — Factor both polynomials here:**
   Numerator: $x^3 - x^2 - 2x =$ _____
   Denominator: $x^3 + 5x^2 + 6x =$ _____

Your Work & Graph:

Summary of features found:

Hole(s): _____ VA: _____

HA: _____ x-int: _____

y-int: _____

Problem 9 (7 points)

The Problem: Use the function $n(x) = -e^x - 5$.
(a) Graph using transformations of $y = e^x$. (b) Write domain and range in interval notation.
(c) Write the equation of the asymptote.

Study Prompts — Answer these before solving!

1. **Topic:** What type of function is this? What overall concept is being tested?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **Parent Function:** Describe the key features of $y = e^x$ (domain, range, asymptote, general shape):

   - Domain: _____ Range: _____
   - Asymptote: _____ Shape: _____

4. **Transformations:** Identify each transformation applied to get $n(x) = -e^x - 5$. Write what each part does to the graph:

   - The negative sign in front of e^x: _____
   - The -5 at the end: _____

5. **After Transformations:** What are the domain and range of $n(x)$? What is the new asymptote?

   Domain: _____ Range: _____
   Asymptote: _____

Your Work & Graph:

Domain: _____ Range: _____
Asymptote: _____

Problem 10 (6 points)

The Problem: Aniyah needs to borrow \$11,000 for 6 years. Option A: 4.5% compounded monthly interest. Option B: 3% compounded continuously.
(a) Total interest at 4.5% compounded monthly interest? (b) Total interest at 3% continuous? (c) Which is less?

Study Prompts — Answer these before solving!

1. **Topic:** What two interest formulas are compared in this problem?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **Write the Formulas:** Write each formula and define every variable:

   - **compounded monthly Interest:** $I =$ _____
     Where: $P =$ _____, $r =$ _____, $t =$ _____, $n =$
   - **Continuous Compound:** $A =$ _____
     Where: $P =$ _____, $r =$ _____, $t =$ _____
     To find *interest only*: $I = A - P$

4. **Plug In Values:** Write the values for this problem:

   - $P =$ _____ $t =$ _____
   - Option A: $r =$ _____ Option B: $r =$ _____

Your Work:

Problems 11 & 12 (6 + 3 points)

Problem 11: Write in exponential form.

(a) $\log_5 25 = 2$

(b) $\log\left(\frac{1}{1{,}000{,}000}\right) = -6$

Problem 12: Write in logarithmic form.

$2^6 = 64$

Study Prompts — Answer these before solving!

1. **Topic:** What is the relationship between logarithmic and exponential forms?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **The Conversion Rule:** Write the equivalence that connects logs and exponents. Label base, exponent, and result.

   $$\log_b(N) = E \iff \_\_\_\_\_$$

4. **Special Bases:** What base is assumed when no base is written (as in Problem 11b)?

   _____

5. **Practice the Direction:**

   - Log → Exponential: identify base = _____, exponent = _____, result = _____.
   - Exponential → Log: the base of the power becomes _____.

Your Work:

Problem 13 (6 points)

The Problem: Simplify without a calculator.

(a) $\log_2 512$ (b) $\log\left(\frac{1}{10{,}000}\right)$ (c) $\log \sqrt[4]{1}$ (verify from your exam copy)

Study Prompts — Answer these before solving!

1. **Topic:** Evaluating logarithms by rewriting as _____.
2. **Notes Check:** Page(s)/Section: _____
3. **Key Strategy:** To evaluate $\log_b(N)$, ask yourself: "b to what power equals N?" Write this as an equation you solve:

$$\log_b(N) = x \quad \Longleftrightarrow \quad b^x = N$$

4. **Useful Properties — Fill these in:**
   - $\log_b(b^k) =$ _____
   - $\log(10^k) =$ _____ (common log base 10)
   - $\log\left(\frac{1}{10^k}\right) =$ _____
5. **Rewrite Each:** Before computing, rewrite the argument as a power of the base:
   - $512 = 2$——— so $\log_2 512 =$ _____
   - $\frac{1}{10{,}000} = 10$——— so $\log\left(\frac{1}{10{,}000}\right) =$ _____

Your Work:

Problem 14 (3 points)

The Problem: Use the product property to write $\log_2(8u)$ as a sum of logarithms. Simplify if possible.

Study Prompts — Answer these before solving!

1. **Topic:** Which logarithm property is used here?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **State the Property:**

   $$\log_b(MN) = \_\_\_\_\_$$

4. **Apply & Simplify:** After expanding, can any term be evaluated to an integer? If so, which one and why?

   _____

   _____

Your Work:

Problem 15 (4 points)

The Problem: Write as a sum or difference of logarithms; simplify each term as much as possible.

$$\ln\left(\frac{\sqrt[5]{5}}{rs(uv)^2}\right)$$ (Assume all variables represent positive real numbers.)

Study Prompts — Answer these before solving!

1. **Topic:** Which logarithm properties are needed here (list all that apply)?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **Write All Three Properties:**

   - Product: $\ln(MN) =$ _____
   - Quotient: $\ln\left(\frac{M}{N}\right) =$ _____
   - Power: $\ln(M^p) =$ _____

4. **Rewrite the Radical:** How do you rewrite $\sqrt[5]{5}$ using a rational exponent before applying the power rule?

   $\sqrt[5]{5} = 5$——

5. **Identify your steps:** In what order will you apply the properties? Write your plan.

   Step 1: _____

   Step 2: _____

   Step 3: _____

Your Work:

Problem 16 (5 points)

The Problem: Write as a single logarithm with coefficient 1; simplify as much as possible.

$$4\log_2 r - 2\log_2 s - 7\log_2 t$$

Study Prompts — Answer these before solving!

1. **Topic:** This is the reverse of Problem 15. What are you doing here?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **Strategy — Which property do you apply first?**

   When you see a *coefficient* in front of a logarithm, use the _____ property first:

   $4\log_2 r = \log_2(\_\_\_\_\_)$, $2\log_2 s = \log_2(\_\_\_\_\_)$, $7\log_2 t = \log_2(\_\_\_\_\_)$

4. **Combining:** After applying the power rule, how do addition/subtraction of logs translate into a single log?

   - Addition → _____
   - Subtraction → _____

Your Work:

Problem 17 (4 points)

The Problem: Solve.

(a) $3^x = 81$ (b) $\sqrt{3}^2 = 2^t$

Study Prompts — Answer these before solving!

1. **Topic:** What method is used to solve exponential equations when both sides can/cannot be written with the same base?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **Two Methods:**

   - **Same base method:** If $b^x = b^y$, then _____. Use when? _____
   - **Logarithm method:** Take log of both sides, then use the _____ property to bring down the exponent.

4. **Classify Each Part:** Which method applies to (a)? _____ To (b)? _____

5. **Hint for (b):** Rewrite $\sqrt{3}$ as a power of 3 first: $\sqrt{3} = 3$——

Your Work:

Problem 18 (8 points)

The Problem: Solve. Write the solution set with exact values in terms of common or natural logarithms.

(a) $988 = 11^{y+4}$ (b) $6\log_5(4p-3) - 2 = 16$

Study Prompts — Answer these before solving!

1. **Topic:** What types of equations are (a) and (b)?
   (a) is a(n) _____ equation. (b) is a(n) _____ equation.
2. **Notes Check:** Page(s)/Section: _____
3. **Strategy for (a):** The bases cannot be matched easily.
   - Take log (or ln) of both sides.
   - Apply the _____ property: $\log(11^{y+4}) = (y+4)\cdot\log(11)$.
   - Then isolate y. Write the exact form using logarithms.
4. **Strategy for (b):** The variable is inside the logarithm.
   - First, isolate the logarithm by _____.
   - Then convert to _____ form to remove the log.
   - Finally, solve for p.
5. **Check for Extraneous Solutions (part b):** Why do you need to check your answer(s) when solving logarithmic equations?

   _____

   _____

Your Work:

Problem 19 (7 points)

The Problem: The amount of caffeine remaining after t hours is $C = 100(2)^{-t/6}$.
(a) How long until caffeine drops below 60 mg? (Round to 1 decimal place.)
(b) If Laura can't sleep with more than 50 mg of caffeine, how many hours before bed should she stop drinking coffee? (Round to 1 decimal place.)

Study Prompts — Answer these before solving!

1. **Topic:** What type of application is this? What does the model $C = 100(2)^{-t/6}$ represent?

   _____

2. **Notes Check:** Page(s)/Section: _____

3. **Understand the Model:** Identify each part of $C = 100(2)^{-t/6}$:

   - 100 represents: _____
   - The base 2 with the negative exponent indicates: _____
   - The 6 in the exponent represents the _____ of the caffeine.

4. **Set Up Each Part:**

   - For part (a), substitute $C =$ _____ and solve for t.
   - For part (b), substitute $C =$ _____ and solve for t.

5. **Solving Strategy:** Once you substitute, you have an exponential equation. Outline the steps to isolate t:

   Step 1: _____

   Step 2: _____

   Step 3: _____

Your Work:

Problem 20 (6 points)

The Problem: Solve the system. State if inconsistent or dependent if no unique solution exists.

$$\begin{cases} -3x - 6y = 2 \\ 9x = 4 - 18y \end{cases}$$

Study Prompts — Answer these before solving!

1. **Topic:** What is this topic called, and what methods have you learned to solve these?
   Topic: _____
   Methods: _____
2. **Notes Check:** Page(s)/Section: _____
3. **Three Possible Outcomes:** For a system of two linear equations, there are three possible outcomes. Write what each means geometrically and algebraically:
   - **One solution (consistent/independent):** The lines _____. Algebra gives $x =$ a number.
   - **No solution (inconsistent):** The lines _____. Algebra gives _____.
   - **Infinitely many (dependent):** The lines _____. Algebra gives _____.
4. **Look Ahead:** Rearrange Equation 2 into standard form ($ax + by = c$) below. Do you notice anything when you compare it to Equation 1?
   Eq. 2 rearranged: _____
   Observation: _____

Your Work:

Problem 21 (8 points)

The Problem: Given:

$$C = \begin{pmatrix} 14 & 2 \\ 3 & -\frac{1}{2} \\ 1 & 4 \end{pmatrix} \qquad D = \begin{pmatrix} 3 & 5 & 3 \\ -1 & 6 & 1 \end{pmatrix}$$

Find (if possible): (a) $C + D$ (b) $2C$ (c) CD (d) DC

Study Prompts — Answer these before solving!

1. **Topic:** What is the topic, and what are the dimensions of each matrix?
   Topic: _____
   Dimensions of C: _____ (rows × columns) Dimensions of D: _____
2. **Notes Check:** Page(s)/Section: _____
3. **Rules — Fill in the requirement for each operation:**
   - **Addition ($C + D$):** Requires matrices to have _____ dimensions.
     Is $C+D$ possible here? _____. Why/why not? _____
   - **Scalar Multiplication ($2C$):** Multiply _____.
     Always defined? _____
   - **Matrix Multiplication (CD):** C is $m \times n$, D is $p \times q$. Requires _____ = _____.
     Result size is _____ × _____. Is CD possible? _____ Result size: _____
   - **Matrix Multiplication (DC):** Is DC possible? _____ Result size (if yes): _____

Your Work:

— End of Study Guide —

Last-Minute Checklist before the Exam:

- ☐ I can do polynomial long division without looking at my notes.
- ☐ I know the Remainder Theorem and Factor Theorem from memory.
- ☐ I can list rational zeros using the Rational Zero Theorem.
- ☐ I know the Complex Conjugate Zeros Theorem and can apply it (complex zeros come in pairs).
- ☐ I can find holes, vertical asymptotes, and horizontal asymptotes of rational functions.
- ☐ I can graph a rational function with all features labeled.
- ☐ I can graph exponential functions using transformations.
- ☐ I can switch between logarithmic and exponential forms.
- ☐ I know and can apply all three logarithm properties (product, quotient, power).
- ☐ I can solve exponential and logarithmic equations.
- ☐ I can apply the compounded interest formula and continuous compound interest formulas.
- ☐ I can recognize inconsistent and consistent systems of equations.
- ☐ I know when matrix operations are defined and can carry them out.

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