Day 9 Notes, Wed Feb 26th

Coming up:
- At the end of class Individual quiz on Arithmetic & Geometric Seq / Series
- HW 6 due today by 11:59pm
- HW 7 due Monday
- Written HW 2 will be posted soon
- HW 8 Webassig will be posted today

Recall:

$$3\overline{)15}^{\,5}$$
5 → quotient
15 → dividend
divisor

$15 = 3 \cdot 5$

$$\begin{array}{r} 5 \\ 3\overline{)16} \\ 15 \\ \hline 1 \end{array}$$
1 → remainder

$16 = 3 \cdot 5 + 1$

2.3 Real Zeroes of Polynomial Functions

Next, we are interested in how to get the factored form of higher degree polynomials so that we can get the real zeros fast. We will review, factoring and long division to divide polynomials. Then we will use that to factor higher degree polynomial and interpret the remainder (if any).

Example 1: Divide the polynomial $f(x) = x^3 - 3x^2 + 4$ by $x - 2$ and use the result to factor the polynomial completely.

dividend   divisor

$$\begin{array}{r} x^2 - x - 2 \\ x-2\overline{)\,x^3 - 3x^2 + 0x + 4} \\ x^3 - 2x^2 + 0x + 0 \\ \hline -x^2 + 0x + 4 \\ -x^2 + 2x + 0 \\ \hline -2x + 4 \\ -2x + 4 \\ \hline 0 \end{array}$$

← result quotient

→ dividend

divisor

0 → remainder

Step 0: $f(x) = x^3 - 3x^2 + 0x + 4$

Step 1: ✓

Step 2: $\frac{x^3}{x} = x^2$

Step 3: $x^2(x-2)$
$= x^3 - 2x^2$

Step 4: subtract

Step 5: Repeat

$-x(x-2) = -x^2 + 2x$

$-2(x-2) =$

$x^3 - 3x^2 + 4 = (x-2)(x^2 - x - 2)$

$\boxed{f(x) = (x-2)(x-2)(x+1)}$

$f(x) = (x-2)^2(x+1)$

$x = 2$ $m = 2$ (touch)

$x = -1$ $m = 1$ (cross)

The Division Algorithm

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$\boxed{f(x) = d(x)q(x) + r(x)} \quad \Big| \div d(x)$$

$$d(x) \qquad\qquad d(x)$$

where $r(x) = 0$ *or* the degree of $r(x)$ is less than the degree of $d(x)$. If the remainder $r(x)$ is zero, then $d(x)$ *divides evenly* into $f(x)$.

The division algorithm can also be written as

$$\boxed{\frac{f(x)}{d(x)} = g(x) + \frac{r(x)}{d(x)}}$$

$$\frac{2x^4+4x^3-5x^2+3x-2}{x^2+2x-3} = 2x^2+1 + \frac{x+1}{x^2+2x-3}$$

Example 2: Divide $f(x) = (x^4 - 5x^2 + 4)$ by $(x^2 - 1)$.

$$f(x) = x^4+0x^3-5x^2+0x+4$$

$$\text{divisor} = x^2+0x-1$$

$$\begin{array}{r} x^2 - 4 \\ x^2+0x-1 \overline{\big) x^4+0x^3-5x^2+0x+4} \\ \underline{x^4+0x^3-x^2 \qquad\qquad} \\ -4x^2+0x+4 \\ \underline{-4x^2+0x+4} \\ 0 \end{array}$$

$$\frac{-4x^2}{x^2} = -4$$

$$x^4-5x^2+4 = (x^2-1)(x^2-4) = (x-1)(x+1)(x-2)(x+2)$$

Example 3: Divide $f(x) = -2 + 3x - 5x^2 + 4x^3 + 2x^4$ by $x^2 + 2x - 3$

$$
\begin{array}{rl}
 & \boxed{2x^2+1} \quad \text{quotient} \\
\cline{2-2}
x^2+2x-3 & \big)\; 2x^4+4x^3-5x^2+3x-2 \\
 & \; 2x^4+4x^3-6x^2 \\
\cline{2-2}
 & \qquad\qquad\qquad x^2+3x-2 \\
 & \qquad\qquad\qquad x^2+2x-3 \\
\cline{2-2}
 & \qquad\qquad\qquad \boxed{x+1} \\
 & \qquad\qquad\qquad \text{remainder}
\end{array}
$$

$$\frac{2x^4+4x^3-5x^2+3x-2}{x^2+2x-3} = 2x^2+1 + \frac{x+1}{x^2+2x-3}$$

or

$$2x^4+4x^3-5x^2+3x-2 = (2x^2+1)(x^2+2x-3) + x+1$$

Group Practice: Do @ home as written HW2

1. Divide the polynomial $f(x) = (x^3 - 9x^2 + 26x - 24)$ by $d(x) = (x - 2)$.
   (a) Does $x - 2$ divide evenly in $f(x)$? Yes
   (b) Calculate the value of $f(2)$.

$$x^2 - 7x + 12$$
$$x-2 \overline{)\, x^3 - 9x^2 + 26x - 24}$$
$$x^3 - 2x^2$$
$$-7x^2 + 26x - 24$$
$$-7x^2 + 14x$$
$$12x - 24$$
$$12x - 24$$
$$\boxed{0} \quad r=0$$

$$f(2) = 2^3 - 9\cdot 2^2 + 26\cdot 2 - 24 = 8 - 36 + 52 - 24 = 0$$

2. Divide $f(x) = (x^4 - 5x^3 + 2x^2 + 8x - 6)$ by $d(x) = (x - 3)$
   (a) Does $x - 3$ divide evenly in $f(x)$?
   (b) Calculate the value of $f(3)$.

$$f(3) = -18$$

$$x^3 - 2x^2 - 4x - 4$$
$$x-3 \overline{)\, x^4 - 5x^3 + 2x^2 + 8x - 6}$$
$$x^4 - 3x^3$$
$$-2x^3 + 2x^2 + 8x - 6$$
$$-2x^3 + 6x^2$$
$$-4x^2 + 8x - 6$$
$$-4x^2 + 12x$$
$$-4x - 6$$
$$-4x + 12$$
$$\boxed{-18} \leftarrow r$$

The Remainder and Factor Theorem

Remainder Theorem:
If a polynomial $f(x)$ is divided by $x - a$, then the remainder is equal to $f(a)$.

Factor Theorem:
A polynomial $f(x)$ $\underbrace{\text{has a factor } (x - a)}_{\text{divides evenly}}$ if and only if $f(a) = 0$.