Math 140: Tuesday, Oct 29, Day 16 Notes

Agenda for Tuesday, Oct 29, 2024	
Chapter 3 Probability	**1**

Coming up

! Online HW 9 due today

Discrete Random variables

Example 1. Consider tossing a coin 3 times. The sample space for the toss of 3 coins is

S={TTT, THH, HTH, HHT, HTT, THT, TTH, HHH}

Lets say we are interested in the **number of heads** you get when you toss the coin 3 times.

X = the number of heads that appear when you perform the experiment

It is called a random variable.

What are the possible values for X?

X = {0, 1, 2, 3}

Make a table with all the possible values of X and their corresponding probabilities.

X	P(X)
0	1/8
1	3/8
2	3/8
3	1/8

Recall: weighted mean

90	45%
80	15%
⋮	

$= 90(.45) + 80(.15) + \dots$

We call X a **random variable** and the table is called a **probability distribution function(pdf)**.

Definitions:
Random Variable – a variable whose numeric value is determined by outcome of a random experiment. We have two types of random variables:

1. Continuous random variables X: results of measurements (take on any real number : 1.01, 1.00911)

2. Discrete random variables X: usually results of "counts"
   - take on a countable number of distinct values
   -2, -1, 0, 1, 2, 3 (count them : 6 values)

Probability distribution function (pdf)– lists the probabilities for each outcome of the random variable:

We use capital letter for random variables: X, Y, Z.

Example 2. Determine if the following random variables is continuous or discrete.
X: length of time it takes to get to work → continuous
Y: number of tornadoes in the month of June → discrete

X | $P(X)$

Rules of a probability distribution:

1. each probability is between 0 and 1
2. all probabilities add to 1.

Example 3. Find the missing probabilities that will make this distribution a proper probability distribution.

X: number of policies sold by insurance salespeople in a day.

x	0	1	2	3	4	5	6
P(x)	0.05	? .15	0.23	0.21	0.17	0.11	0.08

add up to .85

- Find P(x=1) $= 1 - .85 = .15$

- Find P($x > 4$) and interpret it.

$P(x>4) = P(x=5) + P(x=6) = .11 + .08 = .19$

The chance of a salesperson selling or more than 4 policies in a day is 19%..

Example 4. Create a probability distribution where X is the resulting sum in rolling a pair of standard dice.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

36

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$P(\text{sum} \geq 10) = P(X \geq 10) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{6}{36} = 1/6$$

Mean, Expected Value and ~~Standard Deviation~~

X	$P(X)$
x_1	$P(x_1)$
x_2	$P(x_2)$
$\vdots$	$\vdots$
x_n	$P(x_n)$

Let X be a random variable and $x_1, x_2, \cdots x_n$ be the list of possible outcomes for X.

- Expected Value $E(X)$ or mean μ of a discrete probability distribution:

$$\mu = E(X) = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_2 P(x_n)$$

Do on your own and check answer

Example 5. A men's soccer team plays soccer zero, one, or two days a week. The probability that they play zero days is 0.2, the probability that they play one day is 0.5, and the probability that they play two days is 0.3. Find the long-term average or expected value, μ, of the number of days per week the men's soccer team plays soccer.

X	$P(X)$
0	0.2
1	0.5
2	0.3

$$\mu = E(X) = 0 \cdot (0.2) + (1)(0.5) + (2)(.3)$$
$$= 0 + 0.5 + .6$$
$$= \underline{1.1 \text{ days}}$$

$$\mu = E(X) = 1.1$$

Example 6. A charity organization is selling \$5 raffle tickets as part of a fund-raising program. The first prize is a trip to Mexico valued at \$3450, and the second prize is a weekend spa package valued at \$750. The remaining 10 prizes are \$25 gift cards. The number of tickets sold is 6000. Find the expected value of a single raffle ticket.

	X	$P(x)$
Mexico	3445	1/6000
spa	745	1/6000
gift card	20	10/6000
looser	0-5=-5	5988/6000

$\to L_1$ $\to L_2$

$E(X) = \mu$ = mean (average)

1 Var Stats L_1, L_2 = -4.26

$E(X) = -\$4.26$

Each raffle ticket has an average loss of \$4.26.

So if you buy 100 tickets (100 · \$5 = \$500) expect to lose \$426.

Practice: Check your answers

1. Determine if the following is a probability distribution. If not, determine which property is not met.

   A quality inspector checked for imperfections in rolls of fabric for 1 week. The random variable X represents the number of imperfections found.

x	0	1	2	3	4	5
P(x)	$\frac{3}{4}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{50}$	$\frac{1}{100}$

no → they don't add up to 1.

2. The random variable X represents the number of credit cards that adults have along with the corresponding probabilities. Find the mean (expected value) and standard deviation.

x	0	1	2	3	4	5	6
P(x)	0.06	.58	0.21	0.08	0.05	0.01	0.01

yes

3. A 40-year-old man in the U.S. has a 0.242% risk of dying during the next year . An insurance company charges $275 for a life-insurance policy that pays a $100,000 death benefit. What is the expected value for the person buying the insurance?

-275

X	$P(X)$
99725	.00242
-275	.99758

1-.00242

$E(X) = 99{,}725(.00242) + (-275)(.99758)$

$E(X) = -\$33$