Weighted mean:

Example 1. Your grade in this class is calculated as weighted mean.

Grade Distribution	score	weights
3 In Class Exams 45%	80	.45
Online Canvas Homework 20%	100	.2
Excel Assignments 10%	100	.1
In class group activities 10%	75	.1
Final exam 15%	70	.15

L_1 L_2

1 Var Stats L_1, L_2 = 84

Section 3.2 Measures of variation

In some data sets, the data values are concentrated closely near the mean; in other data sets, the data values are more widely spread out from the mean. We will discuss three measures of variation: the range, the standard deviation, and the variance.

Range: highest value - lowest value

The larger the range, the more data varies.

Standard Deviation: is the average distance a data value is from the mean.

The larger the st. dev., the more the data varies.

The smaller the st. dev, the less the data varies (data is more consistent or more concentrated close to the mean)

Stats section 1 exam scores	Stats section 2 exam scores
49.5, 57, 66, 73, 78, 85, 92	67, 68.5, 71, 72, 74.5, 75
$\bar{x}$ = sample mean = 71.5	$\bar{x}$ = 71.3
σ = sigma = population st.dev σ = 14	σ = 2.9
s = sample st. dev. = 15.1	s = 3.2
Has same units as the data	

Variance: is the square of the standard deviation

variance = (st. dev.)^2

$\sigma^2 = (14)^2 = 196$ population variance $s^2 = (15.1)^2 = 228$ sample variance	$\sigma^2 = (2.9)^2 = 8.4$ $s^2 = (3.2)^2 = 10.2$

Standard Deviation of Grouped Frequency Distribution

M

MPG	Frequency
24 - 31	12
32 - 39	15
40 - 47	8
48 - 55	4
56 - 63	1

L_1: Midpoints of each class

L_2: Frequency

1 Var Stats L_1, L_2

$s = 8.5$ mpg

Try it on your own!

Agenda for Monday, Sep 23rd 2024

Coming up

- ! HW 8 will be posted this evening
- Second **Excel Homework** will be posted Wednesday

Section 3.3 Measures of position

The standard deviation is useful when comparing data values that come from different data sets. If the data sets have different means and standard deviations, then comparing the data values directly can be misleading.

Let x be a data value. A z-score corresponding to x is going to tell how many st. deviations away from the mean that x value is.

Formula: $z_{value} = \frac{value - mean}{st.\ dev.}$

or

$$z_x = \frac{x - \bar{x}}{s}$$

Ex $\bar{x} = 87$, $s = 10$

value = 97

$z_{97} = \frac{97-87}{10} = \frac{10}{10} = 1$ positive z-score means you're above the mean

value = 82

$z_{82} = \frac{82-87}{10} = \frac{-5}{10} = -.5$

negative z-score represent values below mean

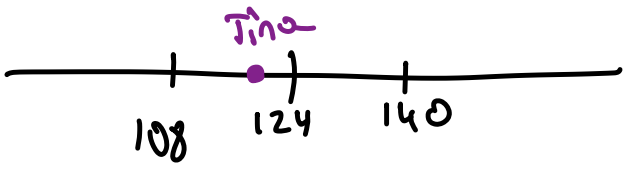
Example 1. The average test score for Mr. Jone's class is 124 with a standard deviation of 16. The average test score for Ms. Siegel's class is 72 with a standard deviation of 8. Tina and Melanie were comparing themselves to see who did better on their prospective exams. Tina scored a 118 in Mr. Jone's exam while Melanie scored a 67 on Ms. Siegel's exam. Find the z-scores and compare.

J

$\bar{x} = 124$

$s = 16$

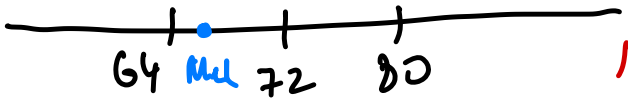
Tina: 118



S

$\bar{x} = 72$

$s = 8$



Mel: $\frac{67 - 72}{8} = -0.63$

$z_{Tina} = \frac{118 - 124}{16} = -0.38$

Tina did better

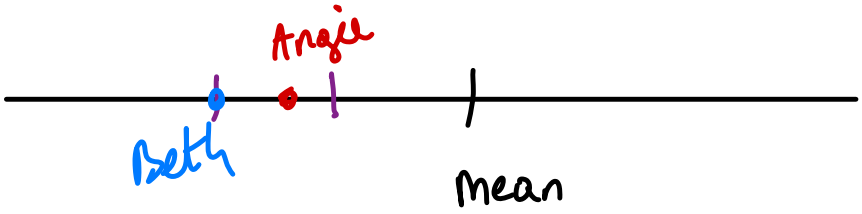
Practice: Two swimmers, Angie and Beth, from different teams, wanted to find out who had the fastest time for the 50 meter freestyle when compared to her team. The table below shows their times.

Swimmer	Time in Seconds	Team Mean Time	Team Standard Deviation
Angie	26.2	27.2	0.8
Beth	27.3	30.1	1.4

Which swimmer had the fastest time when compared to her team?

$z_{Angie} = -1.25$

$z_{Beth} = -2$



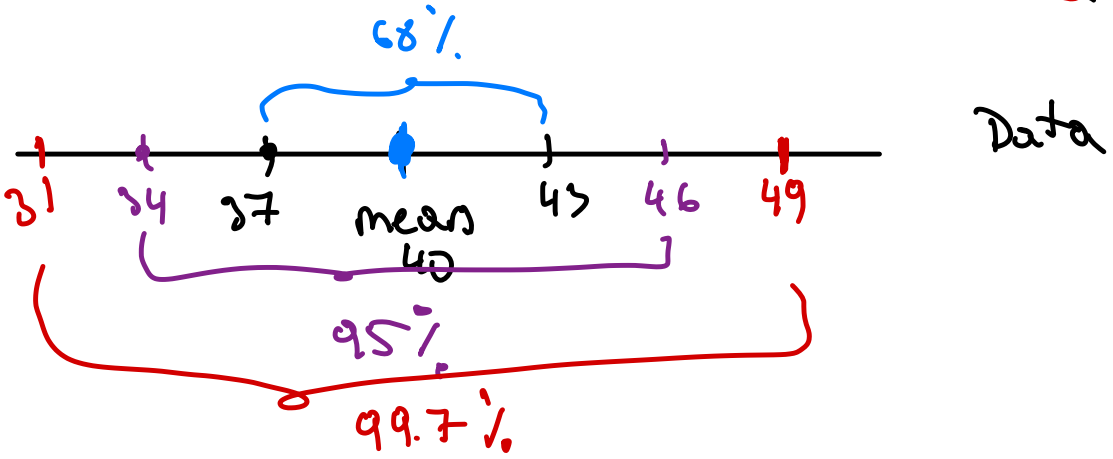
Beth was faster compared to her team.

Empirical Rule

For data having a distribution that is bell-shaped and symmetric:

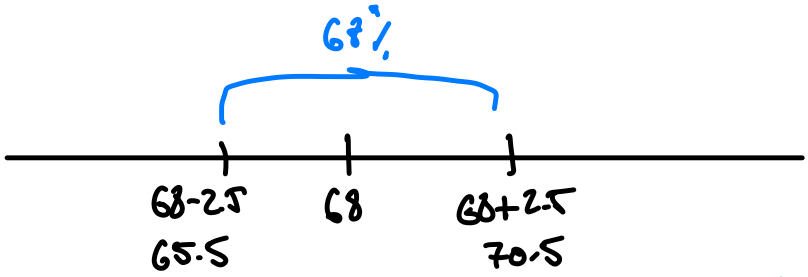
- 68% of the data lies within **one** standard deviation of the mean
- 95% of the data lies within **two** standard deviation of the mean
- 99.7% of the data lies within **three** standard deviation of the mean

Visual of Empirical Rule, use mean = 40 and standard deviation = 3



Example 2. The average height of adult men in the US is 68 inches with a standard deviation of 2.5 inches. Using Empirical Rule

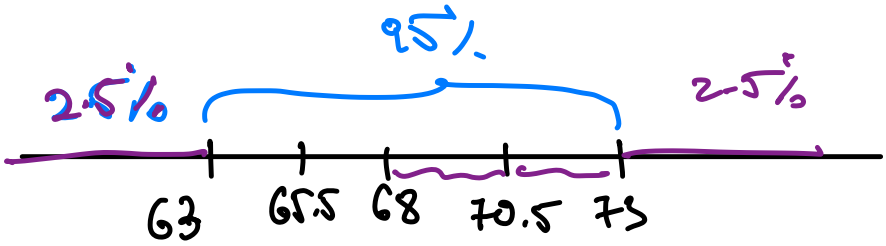
1. Find the heights of the middle 68% of adult men.



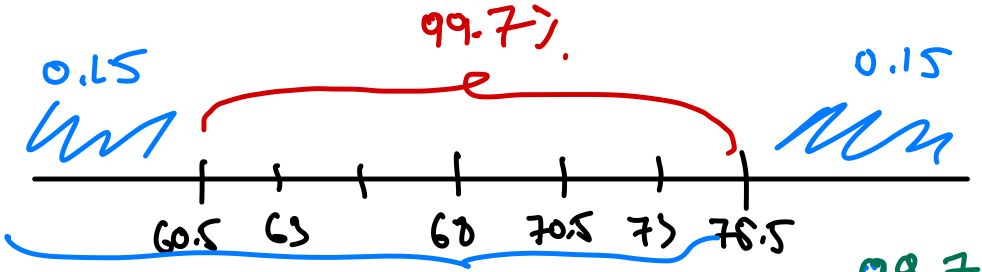
Data : heights of US men

Ans : 68% of men in US have a height between 65.5 and 70.5 inches.

2. What percentage of men is taller than 73 inches?



3. What percentage of men are shorter than 75.5 inches?



100 − 99.7 = 0.3

0.3/2 = 0.15

99.7 + 0.15 = 99.85

4. What is your height if your z-score is 2.5?

74.25?

$2.5 = \frac{x-68}{2.5} \Rightarrow x-68 = (2.5)(2.5)$

$x-68 = 6.25$

$x = 6.25 + 68$

$x = 74.25$ inches