Math 140: Logic: Conditionals

Conditional Statements

The statement "If p, then q." Symbolized by $p \to q$ is called a conditional statement. This is also called <u>implication</u>.
p is called the <u>antecedent (condition)</u>
q is called the <u>consequent (consequence)</u>

Example

If I get my work done by 4 pm, then we will go for a walk.
"I get my work done by 4 pm" is the <u>antecedent</u>.
"We will go for a walk" is the <u>consequent</u>.
There are four possibilities: 1. I get my work done by 4 pm; We go for a walk.
2. I get my work done by 4 pm; We don't go for a walk.
3. I don't get my work done by 4 pm; We go for a walk.
4. I don't get my work done by 4 pm; We don't go for a walk.

When is the promise broke? when the antecedent is true but the consequent is false. $\boxed{T \Rightarrow F}$

Truth Table for the Conditional

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Hidden Conditional Statements

Re-write using the if...then...connective:

1. We will go for a walk if I get my work done by 4 pm.

   If I get my work done by 4 pm, then we will go for a walk

2. Every picture tells a story.

   If it's a picture, then, it tells a story.

3. No guinea pigs are scholars.

   If it's a guinea pig, then it's not a scholar.

4. It is difficult to study when you are distracted.

   If you are distracted, then it's difficult to study.

Symbolic Translation

Given that:

p: I drive to work

q: The dog eats my shoes

r: Milk costs \$3.50 per gallon

Translate into symbols:

1. If I do not drive to work, then milk costs \$3.50 per gallon.

   $\sim p \to r$

2. I drive to work or if the dog does not eat my shoes then milk costs \$3.50 per gallon

   $p \vee (\sim p \to r)$

Relationships Between Conditional Statements

For the conditional $p \to q$:

Converse: $q \to p$

Inverse: $\sim p \to \sim q$

$p \to q$ cond. (implication)

$q \to p$ converse

$\sim p \to \sim q$ inverse

$\sim q \to \sim p$ contrapositive

Contrapositive: $\sim q \to \sim p$

Example

Write the following for the conditional statement: "If you build it, he will come."

$p \to q$

Converse: If he will come, then you build it. $(q \to p)$

Inverse: If you don't build it then, then he won't come $(\sim p \to \sim q)$

Contrapositive: If he doesn't come, then he won't built it. $(\sim q \to \sim p)$

Biconditional

p if and only if q (p is necessary and sufficient for q)

Symbol: $p \leftrightarrow q$

Truth table for the biconditional:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional is true when both statements have the same truth values.

Meaning: $(p \to q) \wedge (q \to p)$

Construct the truth table for the statement:

$$(p\wedge \sim q) \leftrightarrow [(\sim p \vee q) \to p]$$

		C_1	C_2	C_3	C_4	C_5	$C_3 \leftrightarrow C_5$
p	q	$\sim p$	$\sim q$	$p\wedge \sim q$	$\sim p \vee q$	$(\sim p \vee q) \to p$	$(p\wedge \sim q) \leftrightarrow [(\sim p \vee q) \to p$
T	T	F	F	F	T	T	F
T	F	F	T	T	F	T	T
F	T	T	F	F	T	F	T
F	F	T	T	F	T	F	T

$p \to q$ only false when $\boxed{T \to F}$