Math 140: Section 3.7 Discrete Random Variables and Expected Value

Discrete Random variables

Example 1. Consider tossing a coin 3 times. The sample space for the toss of 3 coins is

S={TTT, THH, HTH, HHT, HTT, THT, TTH, HHH }

Lets say we are interested in the **number of heads** you get when you toss the coin 3 times.

X = the number of heads that appear when you do this experimnt

X is called a random variable.

What are the possible values for X? Make a table with all the possible

$X = \{0, 1, 2, 3\}$

values of X and their corresponding probabilities.

X	$P(X)$
0	$P(X=0) = 1/8$
1	$P(X=1) = 3/8$
2	$3/8$
3	$1/8$

$$\mu = E(x) = 0 \cdot \frac{1}{8} + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3 \cdot \frac{1}{8}$$

$$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$= \frac{12}{8} = 1.5$$

We call X a **random variable** and the table is called a **probability distribution function(pdf)**.

Definitions:
Random Variable – a variable whose numeric value is determined by outcome of a random experiment. We have two types of random variables:

1. Continuous random variables X: results of measurements (take on any real value: 1.01, 1.0091)
2. Discrete random variables X: usually the results of counts: 0, 1, 2, 3, 4
   -2, -1, 0, 1, 2, ...

Probability distribution function (pdf)– lists the probabilities for each outcome of the random variable:

Example 2. Determine if the following random variables is continuous or discrete.
X: length of time it takes to get to work continuous
Y: number of tornadoes in the month of June discrete

Rules of a probability distribution:
1. each probability is a number between 0 and 1.
2. all probabilities add up to 1.

X	P(x)
0	1/8
1	3/8
2	3/8
3	1/8

Example 3. Find the missing probabilities that will make this distribution a proper probability distribution.
X: number of policies sold by insurance salespeople in a day.

x	0	1	2	3	4	5	6
P(x)	0.05	?	0.23	0.21	0.17	0.11	0.08

all have to add up to 1

- Find P(x=1) $= 1-(0.05+0.23+0.21+0.17+0.11+0.08) = .15$

prob. that a sales person sells exactly one policy

- Find P($x > 4$) and interpret it.

$$P(X>4) = P(X=5) + P(X=6)$$
$$= 0.11 + 0.08 = 0.19$$

Example 4. Create a probability distribution where X is the resulting sum in rolling a pair of standard dice.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$X \rightarrow$ sum of two numbers on die.

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

P(sum greater or equal to 10) = ?

$$P(X \geq 10) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{6}{36} = 1/6$$

Mean; Expected Value

x_3 x_4
0, 1, 2, 3

Let X be a random variable and $x_1, x_2, \cdots x_n$ be the list of possible outcomes for X.

- Expected Value $E(X)$ or mean μ of a discrete probability distribution:

mean notation : μ (greek letter miu)

or $E(X)$ → expected value of the rand variable X.

$$\mu = E(X) = x_1 P(x_1) + x_2 P(x_2) + \cdots + x_n P(x_n)$$

X # of heads $\mu = E(X) = 0 \cdot \frac{1}{8} +$

Example 5. A men's soccer team plays soccer zero, one, or two days a week. The probability that they play zero days is 0.2, the probability that they play one day is 0.5 and the probability that they play two days is 0.3. Find the long-term average or expected value, μ, of the number of days per week the men's soccer team plays soccer.

X → discrete random variable.

X	$P(X)$
0	0.2
1	0.5
2	0.3

$$\mu = E(X) = 0(0.2) + 1(0.5) + 2(0.3) = 1.1 \text{ days}$$

Example 6. A charity organization is selling $5 raffle tickets as part of a fund-raising program. The first prize is a trip to Mexico valued at $3450, and the second prize is a weekend spa package valued at $750. The remaining 10 prizes are $25 gift cards. The number of tickets sold is 6000. Find the expected value of a single raffle ticket.

X - value of a single raffle ticket
- it's discrete random variable.

	X	$P(x)$
Mexico	3445	1/6000
spa	745	1/6000
gift card	20	10/6000
loose	-5	5988/6000

all possible values of X.

$6000 - \underbrace{(1+1+10)}_{\text{winning}} = 5988$

$\mu = E(X) =$

$= 3445\left(\frac{1}{6000}\right) + 745 \cdot \frac{1}{6000} + 20\left(\frac{10}{6000}\right) + (-5)\frac{5988}{6000}$

$\boxed{E(x) = -\$4.26}$

Each raffle ticket has on average loss of $4.26.

If you buy 100 tickets (100 · $5 = 500) expect to lose $426.

Practice:

1. Determine if the following is a probability distribution. If not, determine which property is not met.

   A quality inspector checked for imperfections in rolls of fabric for 1 week. The random variable X represents the number of imperfections found.

x	0	1	2	3	4	5
P(x)	$\frac{3}{4}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{50}$	$\frac{1}{100}$

No they don't add up to 1.

2. The random variable X represents the number of credit cards that adults have along with the corresponding probabilities. Find the mean (expected value) ~~and standard deviation~~.

x	0	1	2	3	4	5	6
P(x)	0.06	.58	0.21	0.08	0.05	0.01	0.01

$\mu = 0(0.06) + 1(.58) + 2(0.21) + 3(0.08) + 4(0.05) + 5(0.01) + 6(0.01)$

3. A 40-year-old man in the U.S. has a 0.242% risk of dying during the next year . An insurance company charges $275 for a life-insurance policy that pays a $100,000 death benefit. What is the expected value for the person buying the insurance?

X → value of the insurance

X → random variable (you buy insurance and you die :(or don't :)).

so possible values for X are 1) die: $100000 - 275 = 99725$

how much your family gets — how much was paid

2) not die: -275

X	
99725	.00242
-275	.99758

Prob that you live is $1 -$ prob. die $= 1 - .0042$
$= .99757$

$E(X) = 99725(.00242) + (-275)\cdot(.99758)$
$= -\$33$