Math 140: Thursday, Oct 31, Day 17 Notes

Agenda for Thursday, Oct 31, 2024	
Finish Probability !!!	**1**

Coming up

- ! Online HW 10 due Tuesday and Online HW 11 posted today
- Breakfast at Tutoring - Monday, November 4th,8 am - 11 am, Grayslake Tutoring Center (L135)

Binomial Distribution

Next, we will look into a special discrete probability distributions called **Binomial Distributions.**
There are **four conditions** that the experiment has to meet to be considered a **binomial experiment:**

- Condition 1: set number of trials, denoted with n
  ex: toss a coin 10 times ; $n=10$
- Condition 2: each trial has exactly two outcomes
  "success" or "failure"
- Condition 3: each trial is independent
  (trial outcomes don't affect each other)
- Condition 4: probability of success, denoted p, stays the same for each trial.

Example 1. Are the following binomial experiment:

a) Randomly guessing at a multiple-choice question with 4 possible answers has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. Joe guesses on each question with no pattern.

- Condition 1: $n=6$
- Condition 2: ✓
- Condition 3: ✓
- Condition 4: $p = 1/4$

yes!

Q_1:
a)
b)
c)
d)

b) Randomly guessing at a multiple-choice question has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. The first 3 questions have 4 choices, and the last 3 questions have 5 choices each. Cesar guesses on each question with no pattern. Determine if this is a binomial experiment.

- Condition 1: ✓
- Condition 2: ✓
- Condition 3: ✓
- Condition 4: ✗ $p=1/4, \frac{1}{5}$

Q_{1-3}
a)
b)
c)
d)

Q_{4-6}
a)
b)
c)
d)
e)

c) Cyanosis is the condition of having bluish skin due to insufficient oxygen in the blood. About 80% of babies born with cyanosis recover fully. A hospital is caring for 10 babies born with cyanosis. The random variable represents the number of babies that recover fully.

- Condition 1:
- Condition 2:
- Condition 3:
- Condition 4:

Notation and Properties of the Binomial distribution

- The outcomes of a binomial experiment fit a binomial probability distribution.

- We say the random variable $X =$ the # of successes in n trials is a binomial random variable and we write $X \sim B(n, p)$

  x is binomial with n and prob of success p

- To find the probability that our random variable X takes on a certain value (denoted k) we use the following formula:

  $P(x = k) =$ **binomialpdf(n, p, k)** = binomial(10, 0.8, 5) = 0.026

  calculator

  $X \sim B(10, 0.8)$ P(X = 5) → prob that exactly 5 babies recover

- To find the probability that our random variable X takes on valeus less than or equal to a certain value (denoted k) we use the following formula:

  $P(x \leq k) =$ **binomialcdf(n, p, k)**

  $P(X \leq 5) =$ binomialcdf(10, 0.8, 5) = .033

  ↳ cumulative

- The **mean** of a binomial probability distribution is

  average or expected value

  $\mu = np$ ↳ miu

  $\mu = (10)(.8) = 8$

Example 2. Randomly guessing at a multiple-choice question with 4 possible answers has only two outcomes. If a success is guessing correctly, then a failure is guessing incorrectly. Suppose there are 6 multiple choice questions. Mayra guesses on each question with no pattern.

n = 6, p = 0.25

a) Find the probability of Mayra guessing correctly exactly 3 questions.

$P(X = 3) =$ binomialpdf(6, 0.25, 3) = 0.132

k

b) Find the probability of Mayra guessing correctly all of the questions.

$P(X = 6) =$ binomialpdf(6, 0.25, 6) = .00024

k = 6

c) Find the mean or expected number of questions Mayra guesses correctly. $\mu = np = (6)(.25) = 1.5$ question

Example 3. It has been stated that about 41% of adult workers have a high school diploma but do not pursue any further education. 20 adult workers are randomly selected. $p = .41$ $n = 20$

a) Find the probability that **exactly** 2 of them have a high school diploma but do not pursue any further education.

$P(X=2) = \text{binomialpdf}(20, .41, 2) = 0.0024$

b) Find the probability that **at most** 12 of them have a high school diploma.

$P(X \leq 12) = \text{binomialcdf}(20, .41, 12)$

c) Find the probability that **at least** 6 of them have a high school diploma.

$P(X \geq 6) = 1 - \text{binomialcdf}(20, .41, 5)$

d) Find the probability that **more than** 10 of them have a high school diploma.

$P(X > 10) = 1 - \text{binomialcdf}(20, .41, 10)$

e) Find the probability that **less** 5 of them have a high school diploma.

$P(X < 5) = \text{binomialcdf}(20, .41, 4)$

f) How many adult workers do you **expect** to have a high school diploma but do not pursue any further education?

$\mu = np = 20(.41) = 8.2$ people