

Recall: Composition of Functions:

1. One-to-one Functions

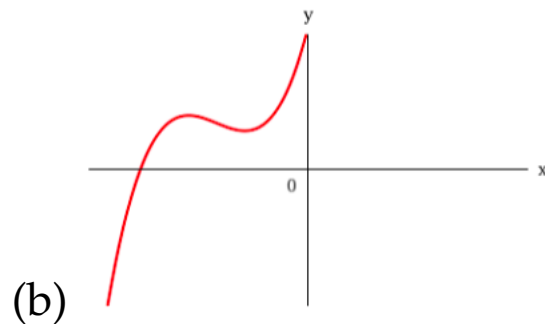
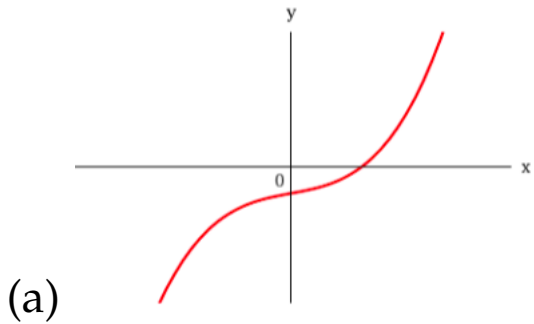
A function f is **one-to-one** if there is a one-to-one correspondence between the inputs and the outputs. In other words, f never takes on the same values twice.

Examples: Is f one-to-one?

$$f(x) = x^2$$

$$f(x) = x^3$$

How to determine if a function f is one-to-one - Graphically
Determine if the functions f and g are one-to-one.



Horizontal Line Test:

Example - Restricted Domain

Graph the function $f(x) = (x + 2)^2 - 1$ and decide if it is one-to-one on its entire domain.

2. Inverse Functions

A one-to-one function with domain A and range B has an **inverse function** which takes on as domain the range of f and gives as output the domain of f . In other words the inverse of a function f undoes what f does. The inverse, denoted as f^{-1} takes as input the output of f and gives as output the input of f , i.e. if $f(x) = y$ then $f^{-1}(y) = x$.

For example if f is the function that takes the input x multiplies it by 5, adds 2 then takes the 5th power of the result, what would the inverse f^{-1} have to be to undo what f did?

The inverse function f^{-1} reverses the effect of f !

INVERSE FUNCTION PROPERTY

Let f be a one-to-one function with domain A and range B . The inverse function f^{-1} satisfies the following cancellation properties:

$$f^{-1}(f(x)) = x \quad \text{for every } x \text{ in } A$$

$$f(f^{-1}(x)) = x \quad \text{for every } x \text{ in } B$$

Conversely, any function f^{-1} satisfying these equations is the inverse of f .

Practice: Are f and g inverses of each other?

$$f(x) = 8 - 4x, \quad g(x) = \frac{8 - x}{4}$$

NOTE: The function and its inverse switch domain and range:

Domain of $f =$

Range of $f =$

3. How to find the inverse of a one-to-one function - Algebraically

Let f be a one-to-one function. Then $f(x) = y$ for any x in the domain of f and f has an inverse denoted f^{-1} .

Example: Let $f(x) = \frac{x - 9}{x + 9}$.

- (a) Graph f . Is f one-to-one?
- (b) Find the domain of f .
- (c) Can you find the range of f without graphing it?
- (d) Find the inverse of f .

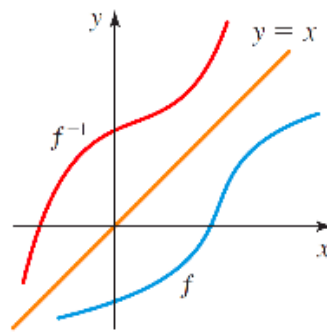
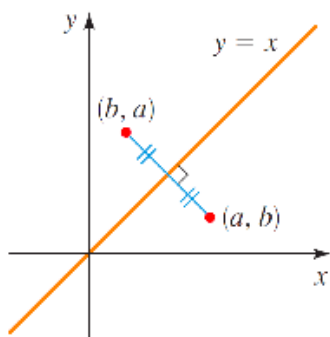
Steps to find inverse functions:

- (e) Can you find the range of f now using the information you have about f^{-1} ?

4. How to find the inverse of a one-to-one function - Graphically

Let f be a one-to-one function. Then $f(x) = y$ for any x in the domain of f and f has an inverse f^{-1} . A point (a, b) on the graph of f is going to become (b, a) on the graph of f^{-1} since the input is switched with the output.

The graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.



Example: Given the graph of f below find the graph of f^{-1} .

