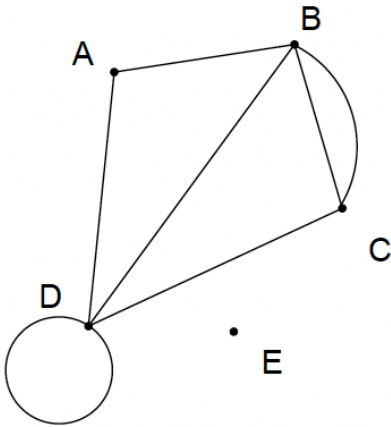


# Math 140: Section 1.1 Intro to Graph Theory: Basic Concepts

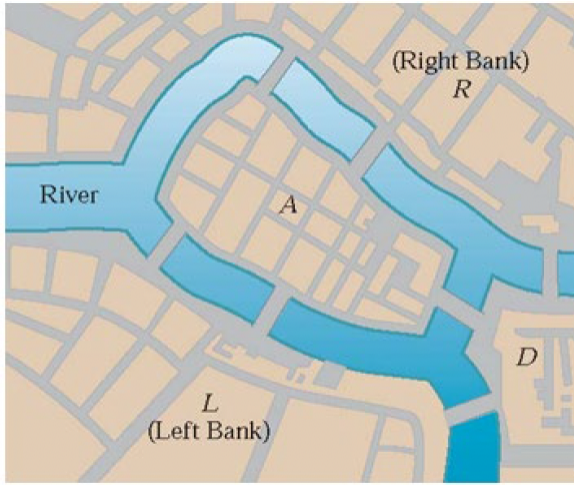
**Graph** - A picture that contains a set of

**Example 1.** Consider the graph below. Give the set of vertices and the set of edges:



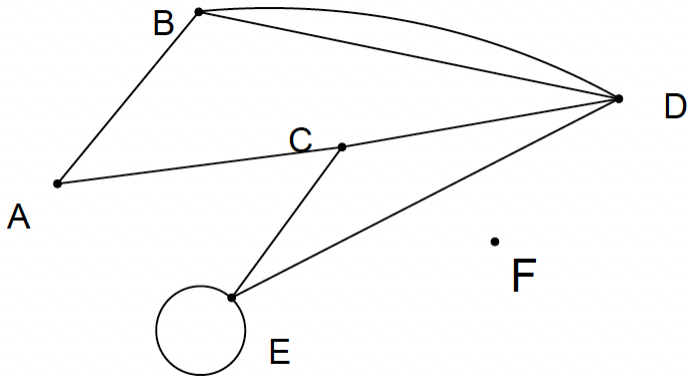
**Example 2.** Draw two different graphs for  $\mathcal{V} = \{L, M, N, P\}$  and  $\mathcal{E} = \{LP, MM, PN, MN, PM\}$

**Example 3.** Back in the 18th century in the Prussian city of Königsberg, a river ran through the city and seven bridges crossed the forks of the river. The river and the bridges are highlighted in the picture to the right. As a weekend amusement, townsfolk would see if they could find a route that would take them across every bridge once and return them to where they started.



Draw the graph modeling this problem.

**Example 4.** For next terminology consider the following graph:



**Adjacent vertices** - vertices connected by an \_\_\_\_\_.

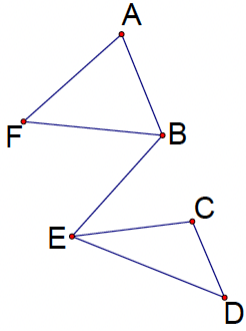
**Adjacent edges** - edges that share a \_\_\_\_\_.

**Degree of a vertex** – the number of \_\_\_\_\_ at that vertex. A loop contributes twice to a degree!

**Path** – a sequence of vertices that describe a trip along the edges of a graph so that you never travel an edge more than once. The **length** of a path is the number of edges in the path.

**Circuit** – a path that starts and ends at the \_\_\_\_\_ vertex.

**Example 5.** Consider the following graph.



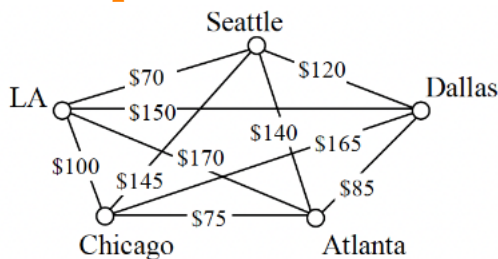
A **bridge** is a

## Weighted Graphs

Depending upon the problem being solved, sometimes weights are assigned to the edges. The weights could represent the distance between two locations, the travel time, or the travel cost. It is important to note that the distance between vertices in a graph does not necessarily correspond to the weight of an edge.

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**Example** Consider the following weighted graph.



The graph shows 5 cities. The weights on the edges represent the airfare for a one-way flight between the cities.

1. How many vertices and edges does the graph have?
2. Is the graph connected?
3. What is the degree of the vertex representing LA?
4. If you fly from Seattle to Dallas to Atlanta, is that a path or a circuit?
5. If you fly from LA to Chicago to Dallas to LA, is that a path or a circuit?