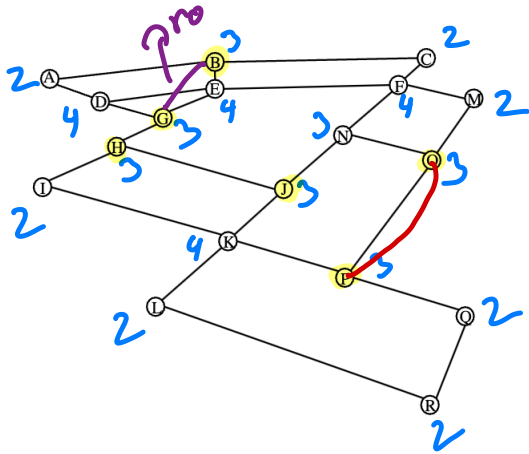


Section 1.2 Continued: Eulerization and Semi-Eulerizations

What do you do if a graph doesn't have an Euler Circuit or an Euler Path but you still need to travel all the edges?

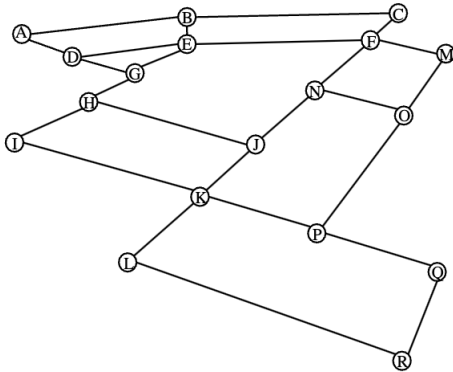
Example 1. Below is a graph that represent a portion of a housing development. As part of her job, the development's lawn inspector has to walk down every street in the development making sure homeowners' landscaping conforms to the community requirements. Naturally, she wants to minimize the amount of walking she has to do.

1. Is it possible for her to walk down every street in this development without having to do any backtracking?



Euler path?
No

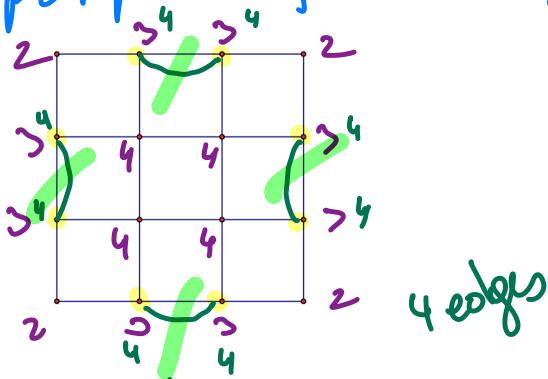
2. Is it possible for her to park her car walk every street once and be back at her car?



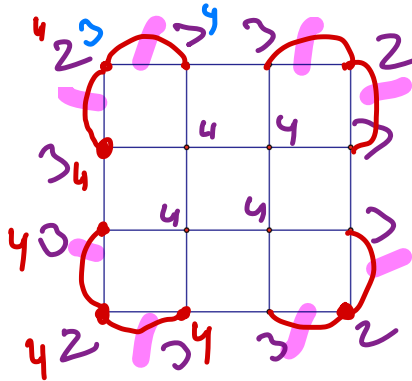
Euler circuit
No because not all
vertices have even
degree?

Eulerization

means creating a new graph (modifying your graph) such that it has an Euler circuit by adding edges to existing edges with the purpose of having no odd vertices.



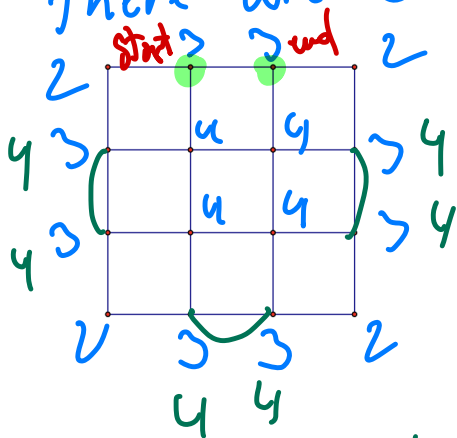
new graph has EC



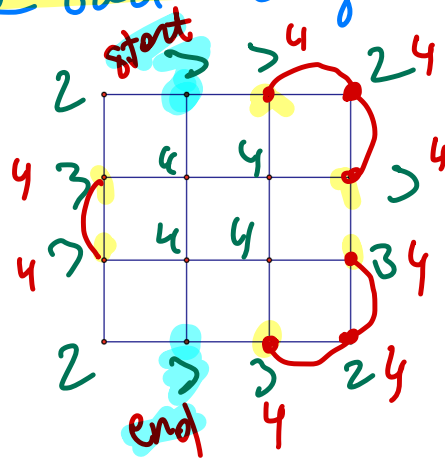
add 8 edges
it's not optimal!

Semi-Eulerization

means creating a new graph (modifying your graph) such that it has an Euler path by adding edges to existing edges such that there are exactly 2 odd degree vertices left.



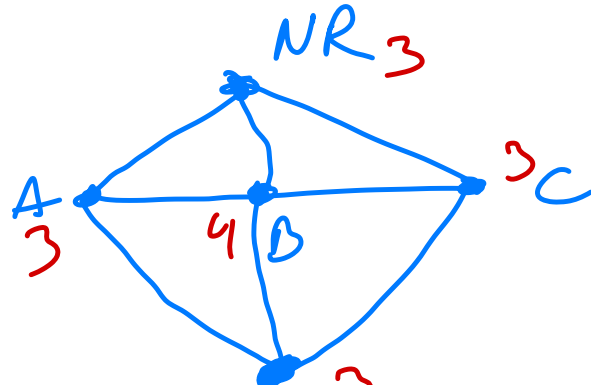
3 edges added
s.t. it has
EP.



5 edges added
s.t. it has
an EP.

Applications of Eulerization and Semi-Eulerization

Example 2. Here is a map of downtown Royalton, showing the Royalton River running through the downtown area and the three islands (A, B, and C) connected to each other and both banks by eight bridges. The Downtown Athletic Club wants to design the route for a marathon through the downtown area.

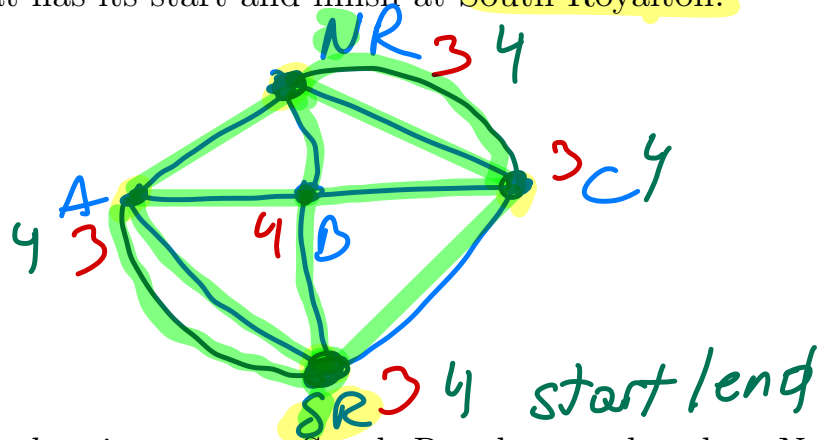


It does not have an EC or EP.

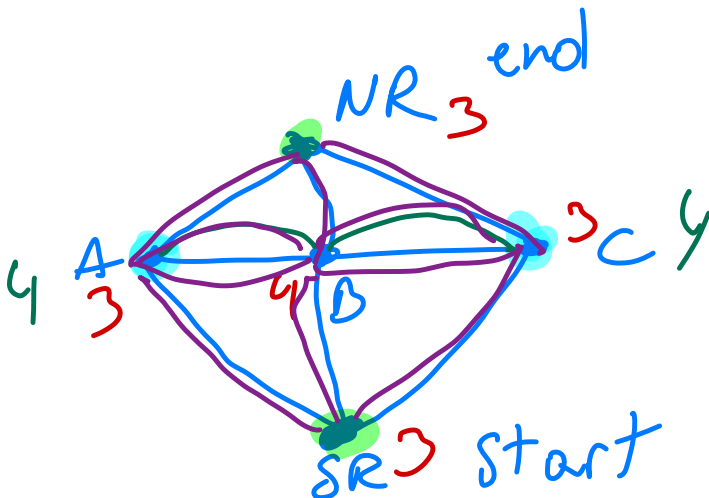
a) Find the circuit for the marathon that has its start and finish at South Royalton.

Eulerize the graph

SANCSABNCBS



b) Find the path for the marathon that has its start at South Royalton and end at North Royalton.



SABCBANCSBN