Monday, April 21st

Finished Graphs of Sin & Cosine
& Tangent, cotangent
Secand and Cosecant.

Horizontal translations:

The constant c in the general equations

$$y = a\sin(bx - c)$$

and

$$y = a\cos(bx - c)$$

creates *horizontal translations* (shifts) of the basic sine and cosine curves. Comparing $y = a\sin bx$ with $y = a\sin(bx - c)$, you find that the graph of $y = a\sin(bx - c)$ completes one cycle from $bx - c = 0$ to $bx - c = 2\pi$.
By solving for x, you can find the interval for one cycle to be

$bx - c = 0$ & $bx - c = 2\pi$ $\quad$ $1 + c$

$\quad +c \quad +c$ $\qquad$ $bx = 2\pi + c$ $\quad$ $1 \div b$

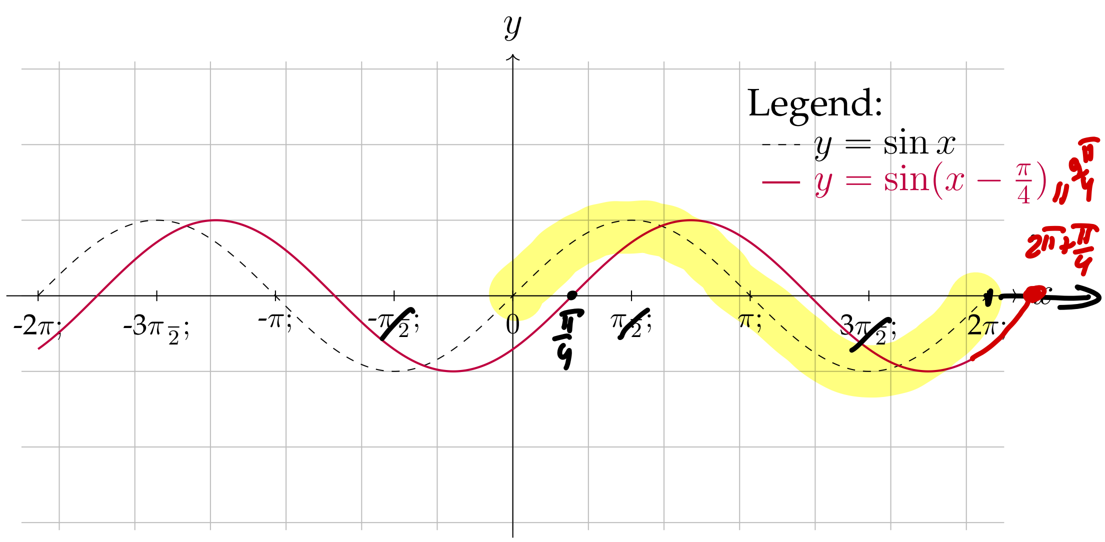
$bx = c$ $\qquad$ $x = \frac{2\pi}{b} + \frac{c}{b}$ or $\boxed{x = \frac{c}{b} + \frac{2\pi}{b}}$

$\boxed{x = \frac{c}{b}}$

$$\underbrace{\frac{c}{b}}_{\text{Left endpoint}} \le x \le \underbrace{\frac{c}{b} + \frac{2\pi}{b}}_{\text{Right endpoint}}$$

Period

This implies that the period of $y = a\sin(bx - c)$ is $2\pi/b$, and the graph of $y = a\sin bx$ is shifted by an amount c/b. The number c/b is the **phase shift**.



Examples:

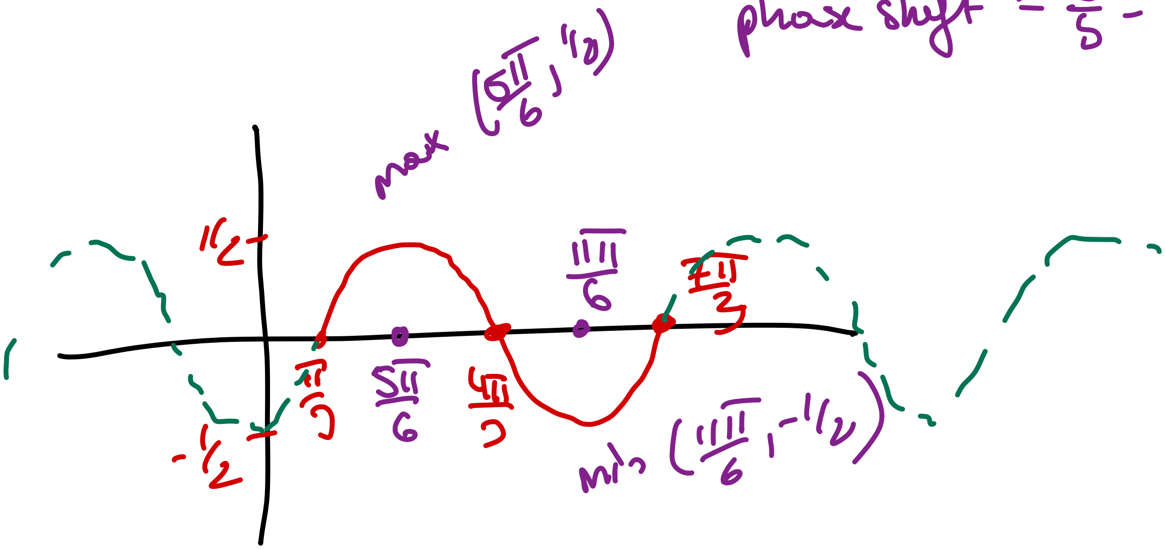
1. Analyze the graph of $y = \frac{1}{2}\sin(1x - \frac{\pi}{3})$.

$(x + \frac{\pi}{3})$ $c = -\pi/3$

$|a| = 1/2$

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$\text{phase shift} = \frac{c}{b} = \frac{\pi/3}{1} = \frac{\pi}{3}$



2. Analyze the graph of $y = -3\,\cancel{\cos}\,\sin(2\pi x + 4\pi)$.

compare with $y = a\sin(bx - c) + d$

$b = 2\pi$, $c = -4\pi$, $d = 0$

$a = -3$

amplitude $= |a| = |-3| = 3$

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$

Left endpoint of one cycle

solve for

$bx - c = 0$

$2\pi x + 4\pi = 0$

$2\pi x = -4\pi$

$x = \frac{-4\pi}{2\pi}$

$x = -2$

Right endpoint of one cycle

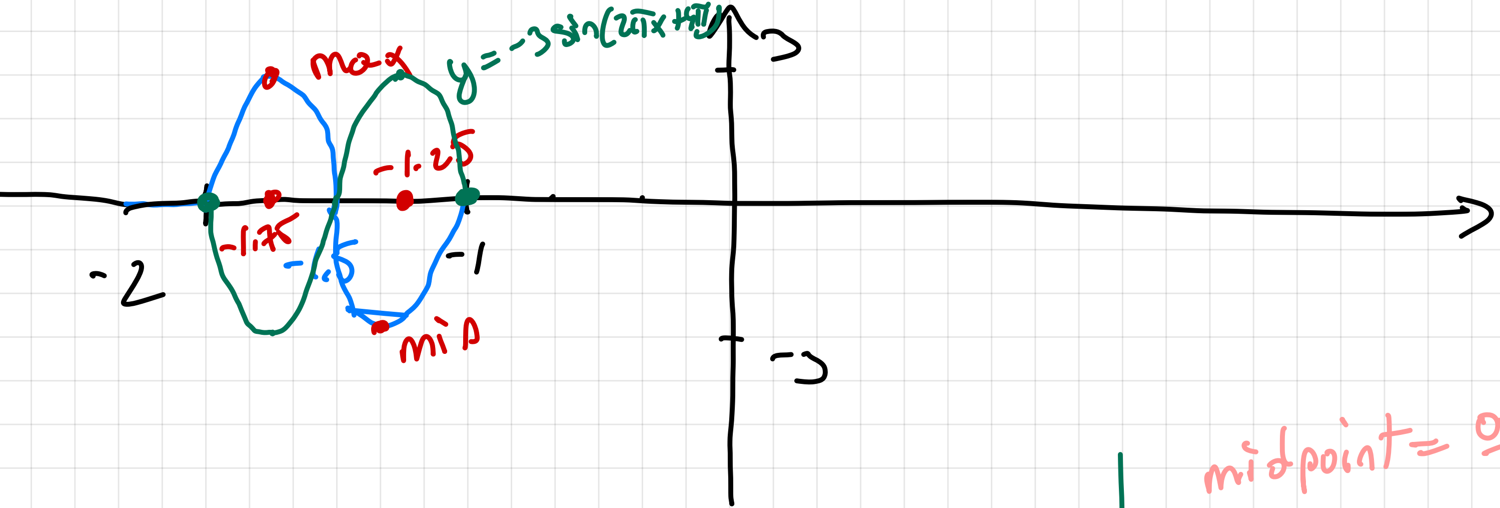
$bx - c = 2\pi$

$2\pi x + 4\pi = 2\pi$

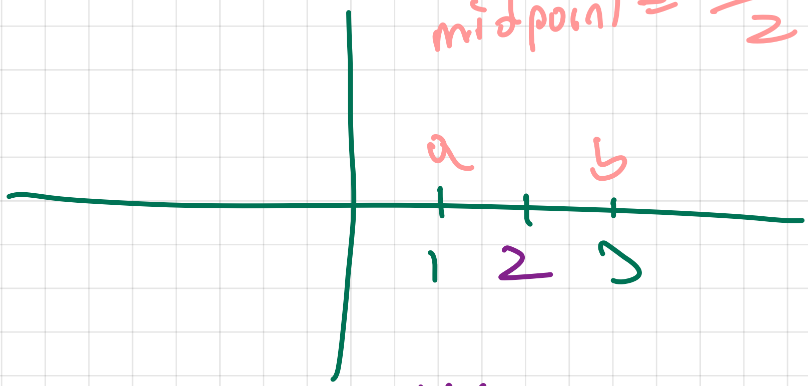
$2\pi x = -2\pi$

$x = \frac{-2\pi}{2\pi}$

$x = -1$



midpoint $= \frac{a+b}{2}$



midpoint setup : $\frac{-2+(-1)}{2} = -\frac{3}{2}$

$= -1.5$

$\frac{-2+-1.5}{2} = -1.75$

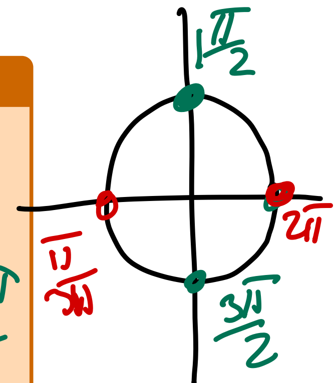
$\frac{-1.5+-1}{2} = -1.25$

$\frac{1+3}{2}$

phase shift $\quad \frac{c}{b} = \frac{-4\pi}{2\pi} = -2$

Section 4.6 Graphs of Other Trig Functions

The Tangent Function



Library of Parent Functions: Tangent Function

Recall:

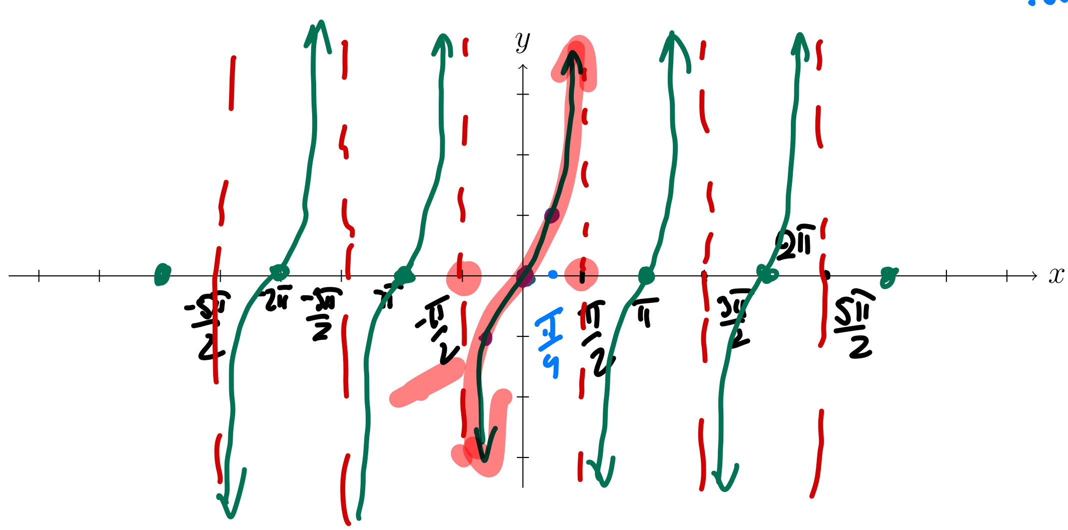
$$\tan x = \frac{\sin x}{\cos x} \qquad \cos x \neq 0$$

$$x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$$

The basic characteristics of the parent tangent function are summarized below.

Domain: all real numbers except odd multiples of $\frac{\pi}{2}$, $x \neq \frac{\pi}{2} + n\pi$, n is integer
Range: $(-\infty, \infty)$
Period: π
x**-intercepts:** $n\pi$, n integer
y**-intercept:** $(0,0)$
Vertical asymptotes:
Odd function: origin symmetry

$\tan\frac{\pi}{4} = 1$



x intercepts: $\tan x = \frac{\sin x}{\cos x} = 0$ when $\sin x = 0$
for multiples of π
$n\pi$, n integer

y intercept: $\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

Sketching the graph of $y = a\tan(bx - c)$ is similar to sketching the graph of $y = a\sin(bx - c)$ in that you locate key points that identify the intercepts and asymptotes. Two consecutive asymptotes can be found by solving the equations $bx - c = -\pi/2$ and $bx - c = \pi/2$ for x. The midpoint between two consecutive asymptotes is an x-intercept of the graph. The period of the function $y = a\tan(bx - c)$ is the distance between two consecutive asymptotes. The amplitude of a tangent function is not defined. After plotting the asymptotes and the x-intercept, plot a few additional points between the two asymptotes and sketch one cycle. Finally, sketch one or two additional cycles to the left and right.

period $= \pi / b$

Example 1 Library of Parent Functions: $f(x) = \tan x$

Sketch the graph of $y = \tan \frac{x}{2}$ by hand. $y = a\tan(bx - c)$

Solution: $a = 1$ $b = \frac{1}{2}$, $c = 0$

Endpoints:

Left: $bx - c = -\frac{\pi}{2}$

$\frac{1}{2}x - 0 = -\frac{\pi}{2}$

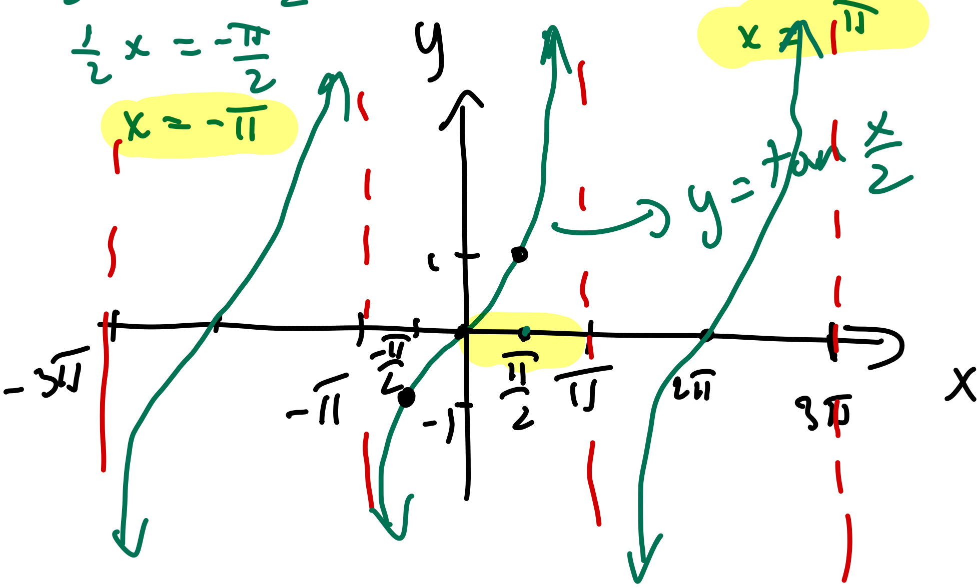
$\frac{1}{2}x = -\frac{\pi}{2}$

$x = -\pi$

Right $bx - c = \frac{\pi}{2}$

$2 \cdot \frac{1}{2}x = \frac{\pi}{2} \cdot 2$

$x = \pi$



$\tan \frac{\pi/2}{2} = \tan\frac{\pi}{4}$
$= 1$

$x = \frac{\pi}{2}$

$\tan\frac{x}{2} = \tan\frac{\pi/2}{2}$

$= \tan\frac{\pi}{4} = 1$

Example 2 **Library of Parent Functions:** $f(x) = \tan x$

Sketch the graph of $y = -3\tan(2x)$ by hand. $a\tan(bx-c)$

Solution: $a=-3$, $b=2$, $c=0$

$\text{period} = \frac{\pi}{b} = \frac{\pi}{2}$

Left:

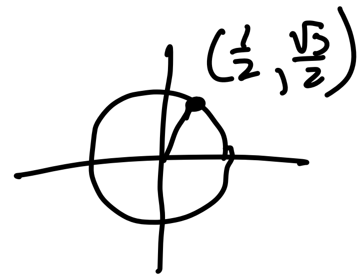
$2x = -\frac{\pi}{2}$

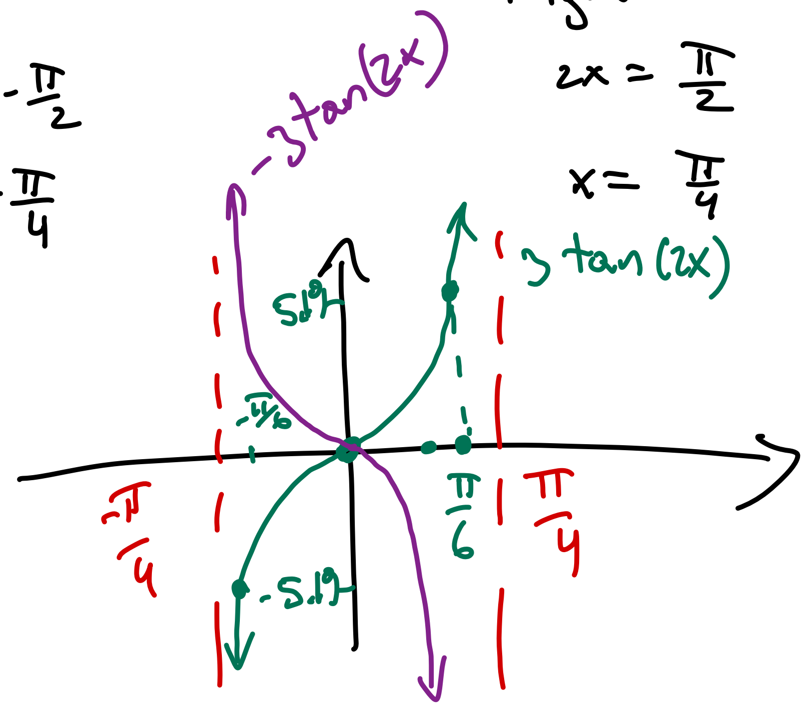
$x = -\frac{\pi}{4}$

Right:

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{4}$



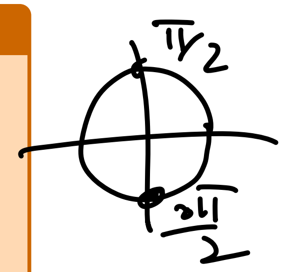


$x = \frac{\pi}{6}$

$3\tan\left(2\cdot\frac{\pi}{6}\right) = 3\tan\frac{\pi}{3}$

$= 3\sqrt{3} = 5.19$

The Cotangent Function



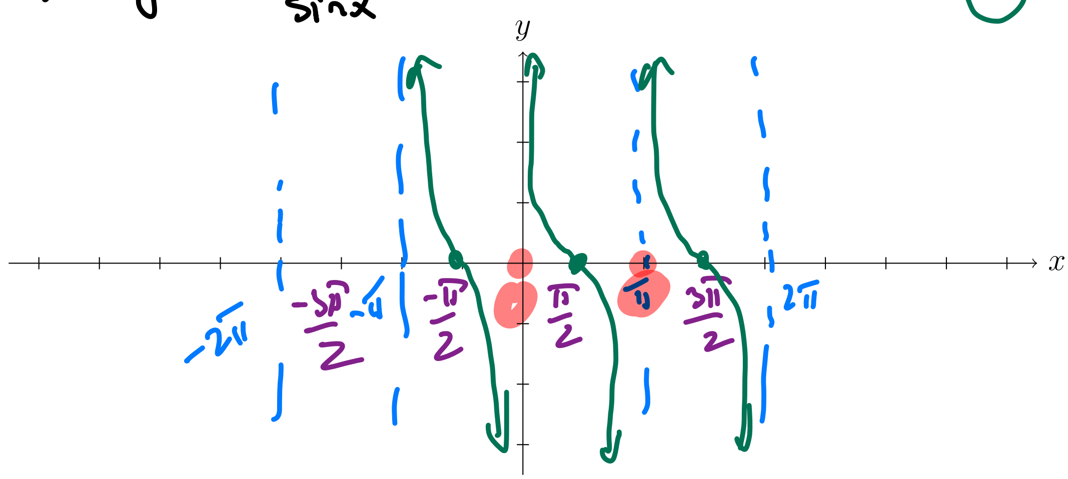
Library of Parent Functions: Cotangent Function

Recall: $\cot x = \dfrac{\cos x}{\sin x}$, $\sin x \neq 0$

The basic characteristics of the parent tangent function are summarized below.

Domain: $x \neq n\pi$ (all real number except multiples of π)
Range: $(-\infty, \infty)$
Period: π
***x*-intercepts:** where $\cos x = 0$, $x = \pm \pi/2, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \dots$
***y*-intercept:**
Vertical asymptotes:
Odd function

y intercept: set $x=0$ $\quad \cot 0 = \dfrac{\cos 0}{\sin 0} = \dfrac{1}{0}$ undefined (no y intercept)
x intercepts: set $y=0$ $\quad \dfrac{\cos x}{\sin x} = 0 \Rightarrow \cos x = 0, \pm\pi/2, \pm\frac{3\pi}{2}, \dots$



Example 3 **Library of Parent Functions:** $f(x) = \cot x$

Sketch the graph of $y = 2\cot\dfrac{x}{3}$ by hand.

$y = a\cot(bx - c)$

Solution:

$a = 2$

$b = \frac{1}{3}$

$c = 0$

period $= \frac{\pi}{b} = \frac{\pi}{1/3} = 3\pi$

Left

$bx - c = 0$

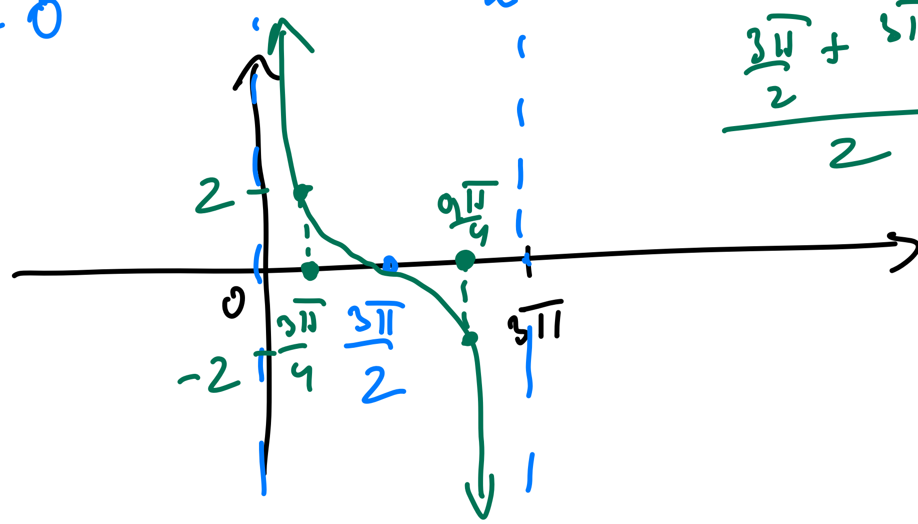
$\frac{x}{3} = 0$

$x = 0$

$bx - c = \pi$

$\frac{x}{3} = \pi$

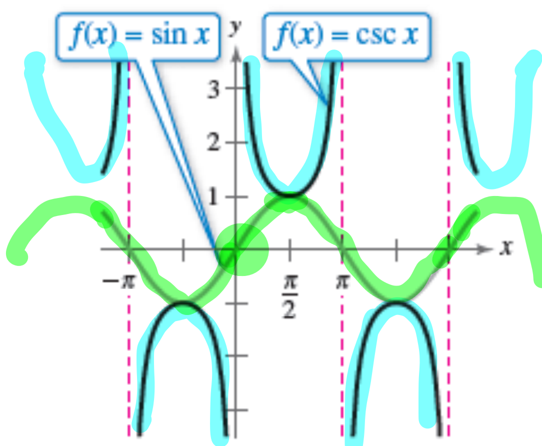
$x = 3\pi$



$$\frac{\frac{3\pi}{2} + 3\pi}{2} = \frac{\frac{3\pi}{2} + \frac{6\pi}{2}}{2} = \frac{9\pi}{4}$$

Graphs of Reciprocal functions

$\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$



Domain: all real numbers x, $x \neq n\pi$

Range: $(-\infty, -1] \cup [1, \infty)$

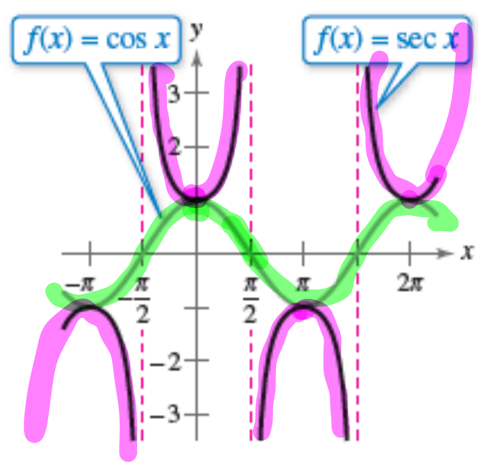
Period: 2π

No intercepts

Vertical asymptotes: $x = n\pi$

Odd function

Origin symmetry



Domain: all real numbers x, $x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

y-intercept: $(0, 1)$

Vertical asymptotes: $x = \frac{\pi}{2} + n\pi$

Even function

y-axis symmetry